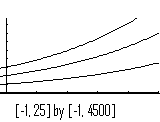
**Differential Equations Application**



**Population Problem**

The population of the little town Scorpion Gulch is now 1000 people. The population is presently growing at about 5% per year. Write a differential equation that expresses population as a function of time. (Get P alsone).

**Punctured Tire Problem**

You have just run over a nail. As air leaks out of your tire, the rate of change of the air pressure inside the tire is directly proportional to that pressure.

a) Write a differential equation that states this fact. Evaluate the proportionality constant if, at time zero, the pressure is 35 psi and decreasing at .28 psi/min.

b) Solve the differential equation subject to the initial condition given in part a.

c) Sketch the graph of the function. Show its behavior a long time after the tire is punctured.

d) What will the pressure be in 10 min after the tire was punctured?

e) The car is safe to drive as long as the tire pressure is 12 psi or greater. For how ling after the puncture will the car be safe to drive?

**Bacteria Problem**

Bacteria in a lab culture grow in such a way that the instantaneous rate of change of the bacteria population is directly proportional to the number of bacteria as a function of time.

a) Write a differential equation that expresses this relationship. Separate the variables and integrate the equation, solve for the number of bacteria as a function of time.

b) Suppose that initially there are 5 million bacteria. Three hours later, the number has grown to 7 million. Write the particular equation that expresses the number of millions of bacteria as a function of the number of hours.

c) Sketch the graph of bacteria versus time.

d) What will the bacteria population be one full day after the first measurement?

e) When will the population reach 1 billion? (1000 million)

**Nitrogen Problem**

When a water-cooled nuclear power plant is in operation, oxygen in the water is transmuted to nitrogen-17. After the reactor is shut down, the radiation from the nitrogen-17 decreases in such a way that the rate of change in the radiation level is directly proportional to the radiation level.

a) Write a differential equation that expresses the rate of change in the radiation level in terms of time.

b) Suppose that when the reactor is first shut down, the radiation level is  units. After 60 seconds the level has dropped to  units. Write the particular equation.

c) Sketch the graph of radiation level versus time.

d) It is safe to enter the reactor compartment when the radiation level has dropped to  units. Will it be safe to enter the reactor compartment 5 min after the reactor has been shut down? Justify your answer.

**Car Trade-In Problem**

Major purchases, like cars, depreciate in value. That is, as time passes, their value decreases. A reasonable mathematical model for the value of an object that depreciates assumes that the instantaneous rate of change of the object’s value is directly proportional to the value.

a) Write a differential equation that says that the rate of change of a car’s trade-in value is directly proportional to that trade in value. Integrate the equation and express the trade-in value as a function of time.

b) Suppose you own a car whose trade-in value is presently $4200. Three months ago its trade-in value was $4700. Find the particular equation that expresses the trade-in value as a function of time since the car was worth $4200.

c) Plot the graph of the trade-in value versus time. Sketch the result.

d) What will the trade-in value be one year after the time the car was worth $4700?

e) You plan to get rid of the car when its trade in value drops to $1200. When will this be?

f) At the time your car was worth $4700, it was 31 months old. What was its trade-in value when it was new?

g) The purchase price of the car when it was new was $16,000. How do you explain the difference between this number and your answer to part f?

**Other Differential Equation Applications**

**Tin Can Leakage Problem**

Suppose you fill a topless can with water, then punch a hole near the bottom with an ice pick. The water leaks quickly at first, then more slowly as the depth of the water decreases. In engineering or physics, you will learn that the rate at which water leaks out is directly proportional to the square root of its depth. Suppose that at time t = 0 min, the depth is 12 cm, and dy/dt is -3 cm/min.

a) Write a differential equation that staets that the instantaneous rate of change of y with respect to t is directly proportional to the square root of y. Find the proportionality constant.

b) Solve the differential equation to find y as a function of t. Use the given information to find the particular solution. What type of function is this?

c) Sketch the graph. Consider the domain of t in which the function gives reasonable answers.

d) Solve the equation from part b algebraically to find the time it takes the can to drain. Compare your answer with the time it would take the can to drain at a constant rate of -3 cm/min.

**Dam Leakage Problem**

A new dam is constructed across Scorpion Gulch. The engineers need to predict the volume of water in the lake formed by the dam as a function of time. At time t = 0 days, the water starts flowing in at a fixed rate F, in cubic feet/hour. Unfortunately, as the water level rises, some leaks out. The leakage rate, L, in cubic feet/hour, is directly proportional to the volume of water, W, in cubic feet, present in the lake. Thus, the instantaneous rate of change of W is equal to F – L.

a). What does L mean in terms of W? Write a differential equation that expresses dW/dt in terms of F, W, and t.

b) Solve for W in terms of t, using initial condition W = 0 when t = 0.

c) The engineers know that water is flowing in at F = 5000 cubic feet/hour. Based on geological considerations, the proportionality constant in the leakage equation is assumed to be .04. Write an equation for W, substituting these quantities.

d) Predict the volume of water in the lake after 10 h, 20 h, and 30 h. After these intervals, how much water has flowed in and how much has leaked out.

e) When will there be 100,000 cubic feet of water in the lake?

f) Find the limit of W as t approaches infinity. State the real-world meaning of this number.

**Sweepstakes Problem**

You have just won a national sweepstakes! Your award is an income of $100 per day for the rest of your life. You decide to put money into a fireproof filing cabinet and let it accumulate. But temptation stets in, and you start spending the money at a rate S, in dollars a day.

a) Let M be the number of dollars you have in the filing cabinet and t be the number of days that you’ve been receiving the money. Assuming that the rates are continuous, write a differential equation that expresses dM/dt in terms of S.

b) Your spending rate, S, is directly proportional to the amount of money, M. Write an equation that expresses this fact, then substitute the result into the differential equation.

c) Separate the variables and integrate the differential equation in part b to get an equation for M in terms of t. For the initial condition, realize that M = 0 when t = 0.

d) Suppose that each day you spend 2% of the money in your filing cabinet, so the proportionality constant in the equation for S is .02. Substitute this value into the equation you found in part c to get M explicitly in terms of t.

e) Sketch the graph.

f) How much money will you have in the filing cabinet after 30 days, 60 days, and 90 days? How much has come in, how much have you spent?

g) How much money do you have in the filing cabinet after one year? At what rate is the amount increasing at this time?

h) What is the limit of M as t approaches infinity?

**Hot tub Problem**

At time t = 0 the drain on a cylindrical hot tub 8ft wide and 4 ft deep is opened and water flows out. The rate at which it flows is proportional to the square root of the depth, y, in feet. Because the tub has vertical sides, the rate is also proportional to the square root of the volume, V, in cubic feet, of the water left.

a) Write a differential equation for the rate at which water flows from the tub. That is, write an equation for dV/dt in terms of V.

b) Separate the variables and integrate the equation you wrote in part a. Transform the result so that V is expressed explicitly in terms of t. Tell how V varies with t.

c) Suppose the tub initially contains 196 cubic feet of water and that when the drain is first opened the water flows out at 28 cubic feet/min (when t = 0, dV/dt = -28). Find the particular solution of the differential equation that fits these initial conditions.

d) Naïve thinking suggests that the tub will be empty after 7 minutes since it contains 196 ft and the water is flowing out at 28 ft/min. Show this conclusion is false and justify your answer.

e) Does the mathematical model predict a time when the tub is comp[lately empty, or does the volume, V, approach zero asymptotically? If there is a time, state it.

f) Draw a graph.